Leçon 8.2
Oscillateur Généralisé Conservatif
– Exemples –

ME-332 – Mécanique Vibratoire

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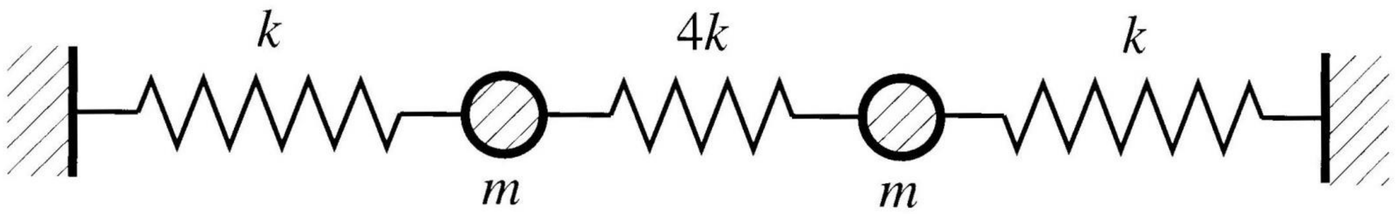
École polytechnique fédérale de Lausanne

Introduction

- Système 2DdL
- Poutre avec 2 masses
- Quotient de Rayleigh
- 3 masses en mode libre-libre

Exemple 8.1: Système à 2 DdL



$$m = 0,1 \text{ kg} \qquad k = 43,865 \text{ N / m}$$

$$\boldsymbol{x}(t=0) = \boldsymbol{X}_0 = \{20,0\}^T \text{ mm}$$

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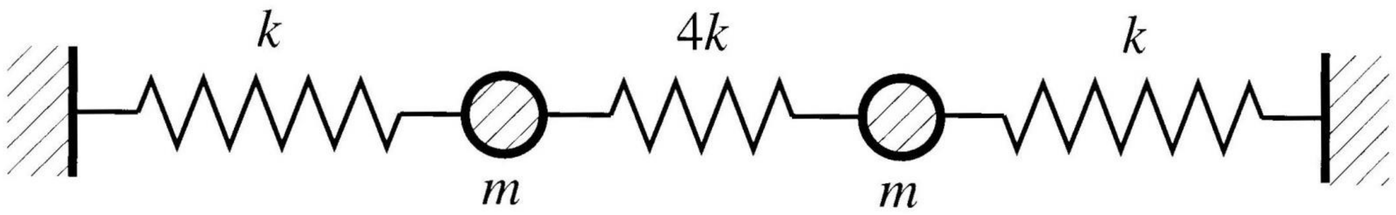

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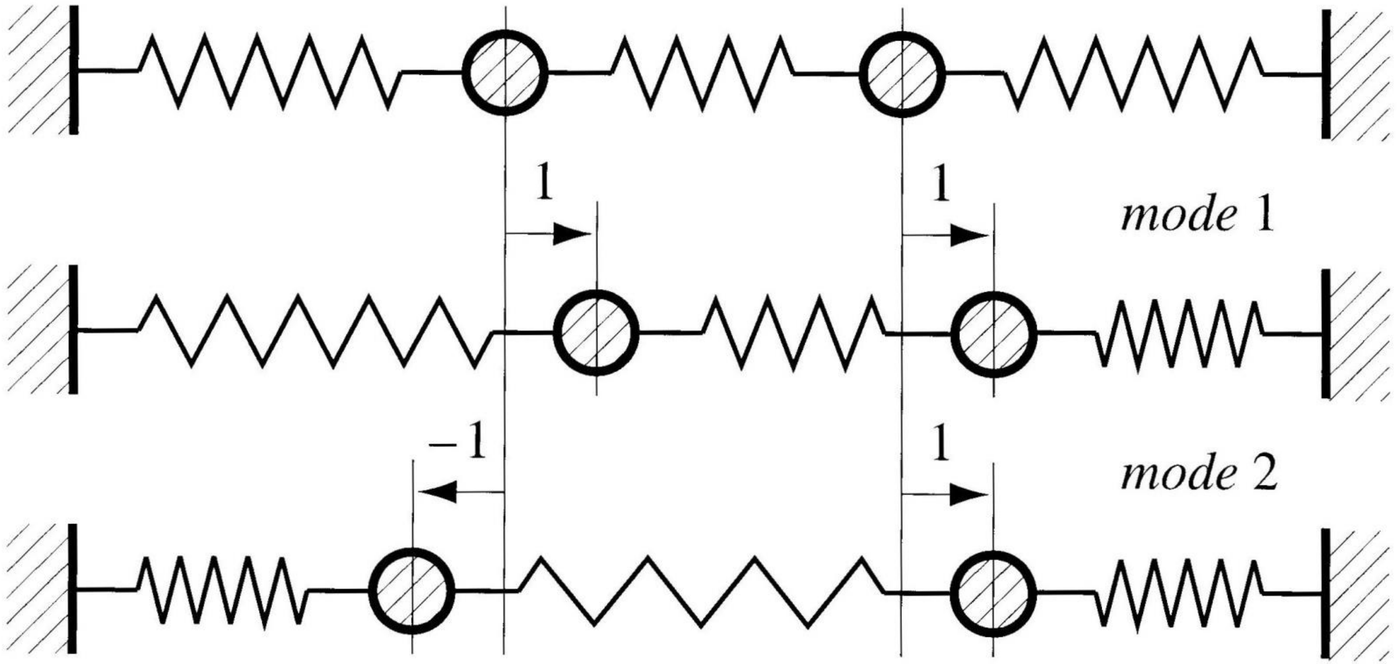

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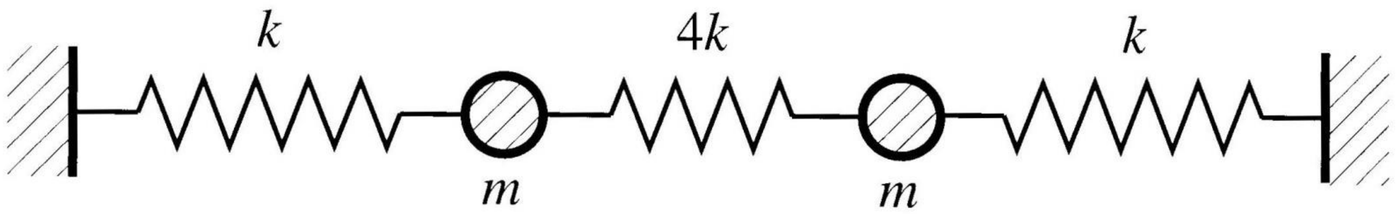

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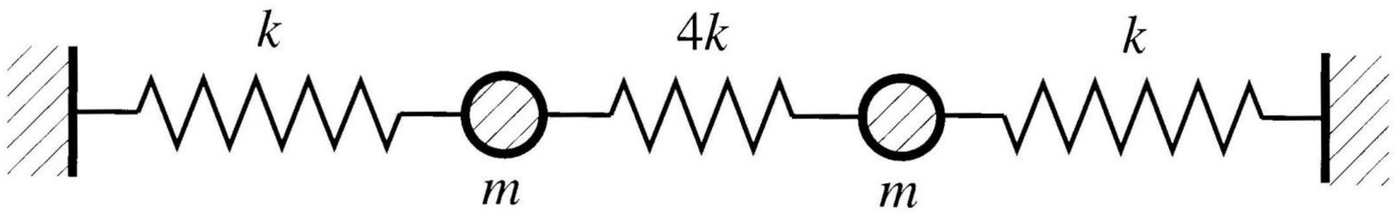

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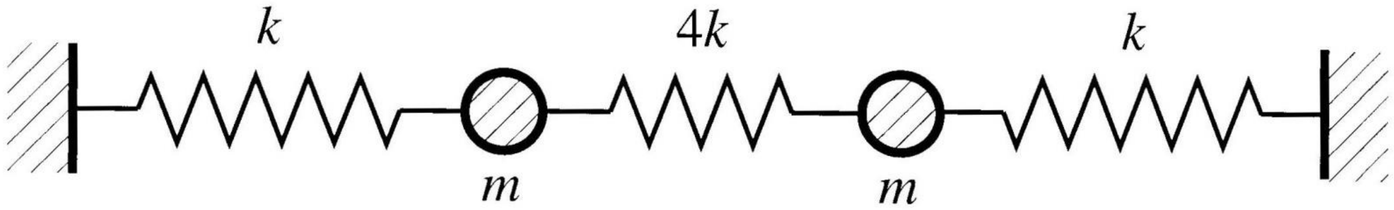
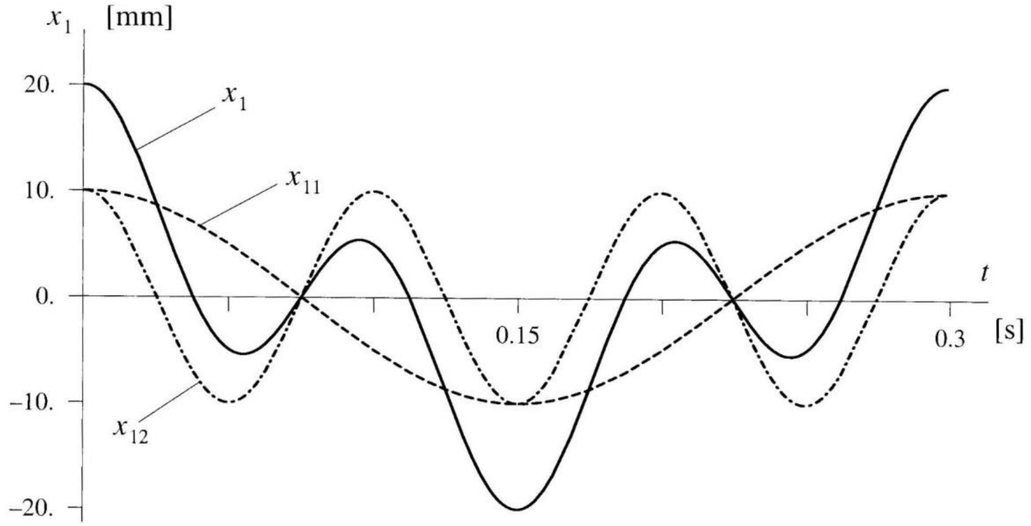
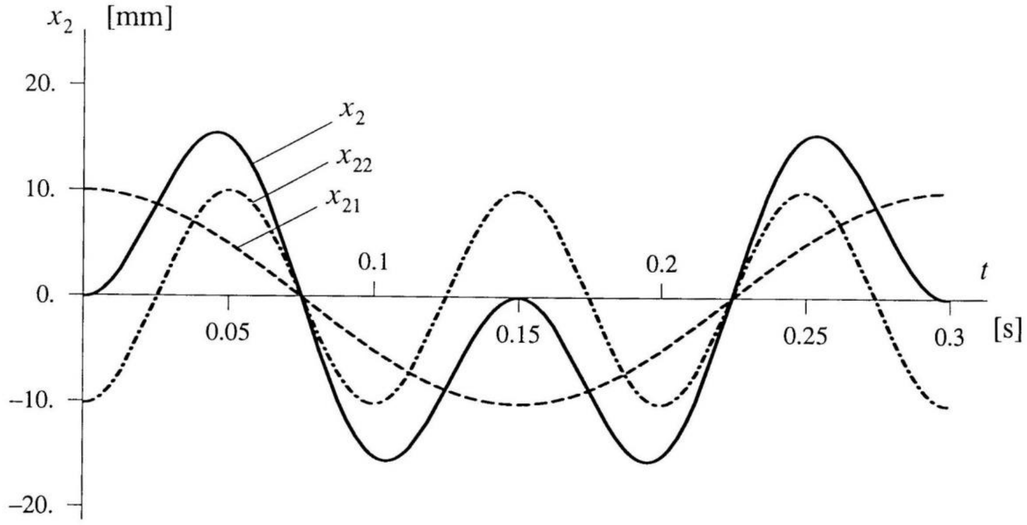
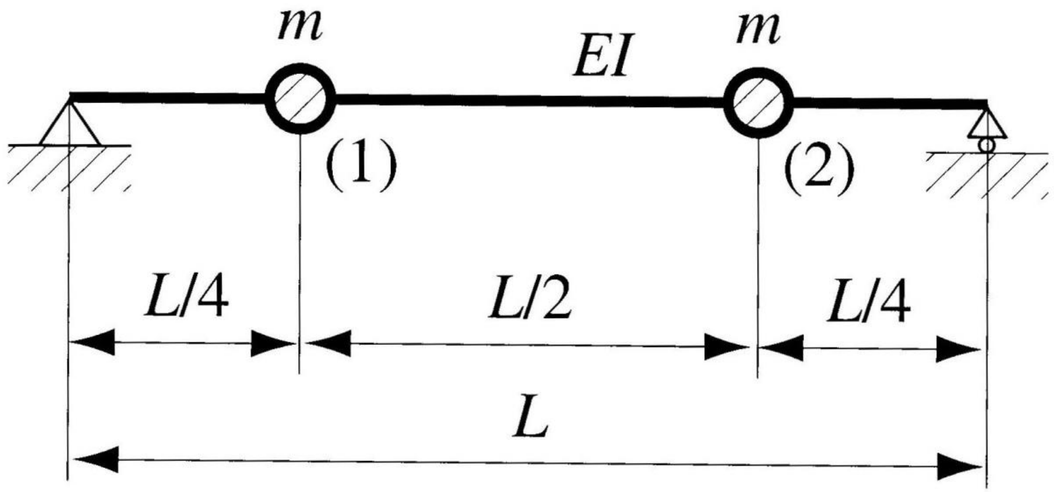






Exemple 8.2: Poutre supportant des masses



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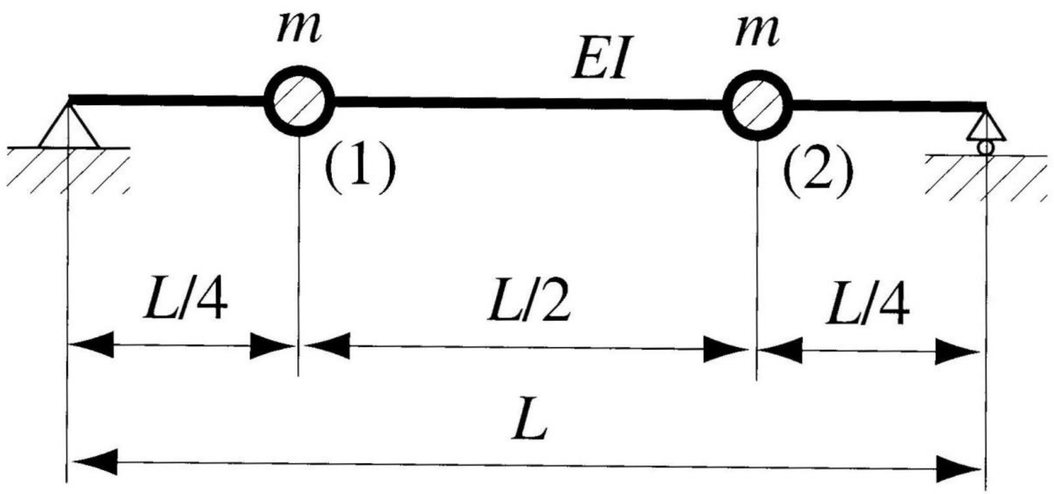

Régime libre associé aux *vibrations latérales d'une poutre* de flexibilité $[\alpha]$ supportant des masses concentrées

$$[K]^{-1}[M]\ddot{\boldsymbol{x}} + \boldsymbol{x} = \boldsymbol{0}$$

$$[\alpha][M]\ddot{\boldsymbol{x}} + \boldsymbol{x} = \boldsymbol{0}$$

$$[E]\ddot{\boldsymbol{x}} + \boldsymbol{x} = \boldsymbol{0}$$

Matrice de masse de la poutre

$$[M] = m[I]$$

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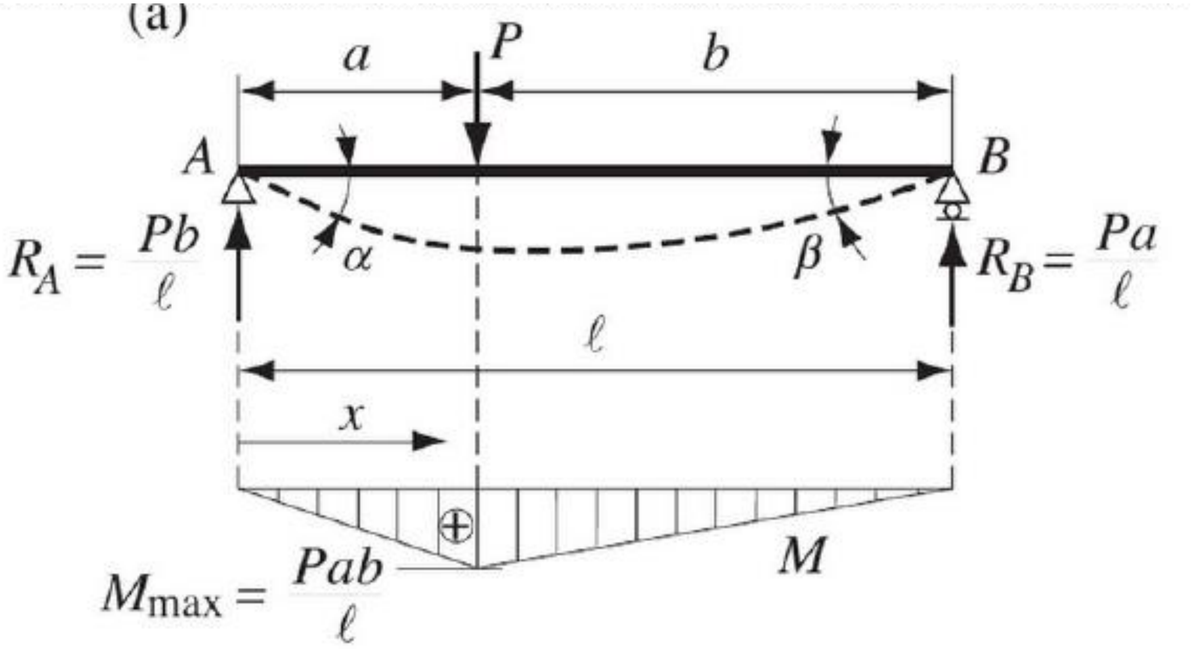

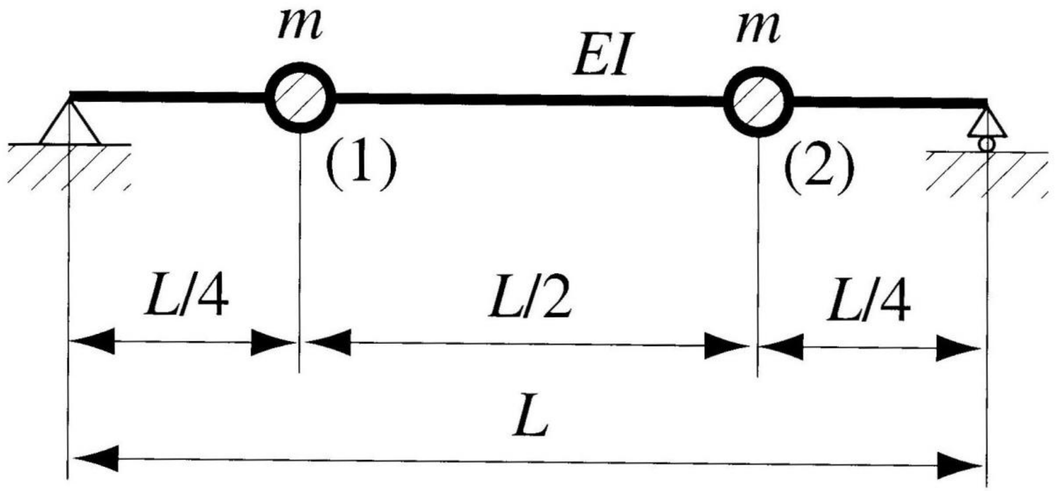

$$y = \frac{P}{6\ell EI}(2ab^2x + a^2bx - bx^3)$$

$$(0 \le x \le a)$$

$$y = \frac{P}{6\ell EI}[2a^2b(\ell - x) + ab^2(\ell - x) - a(\ell - x)^3] \quad (a \le x \le \ell)$$

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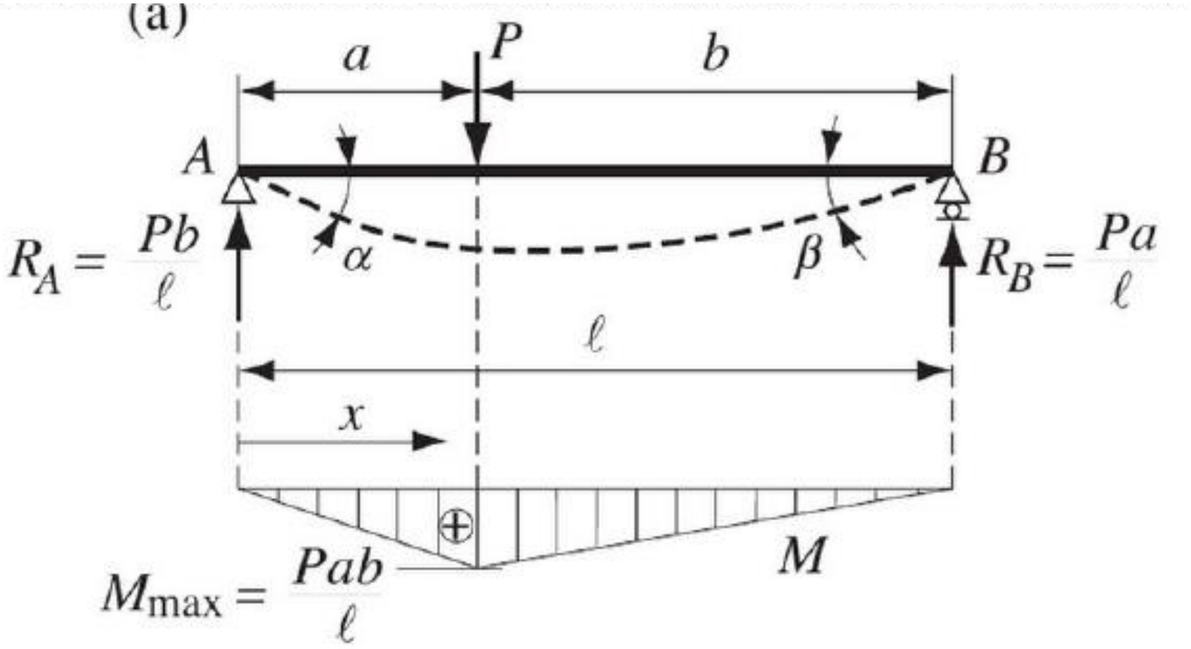

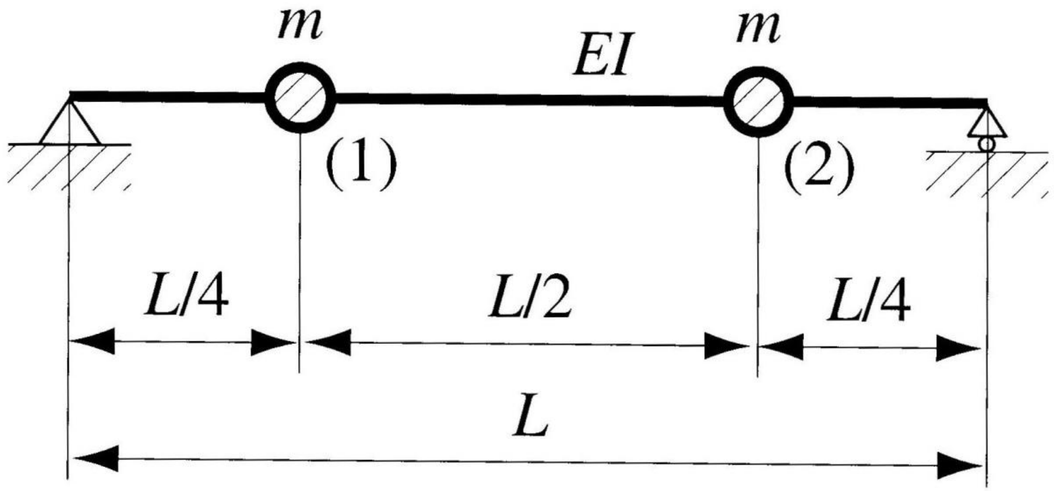

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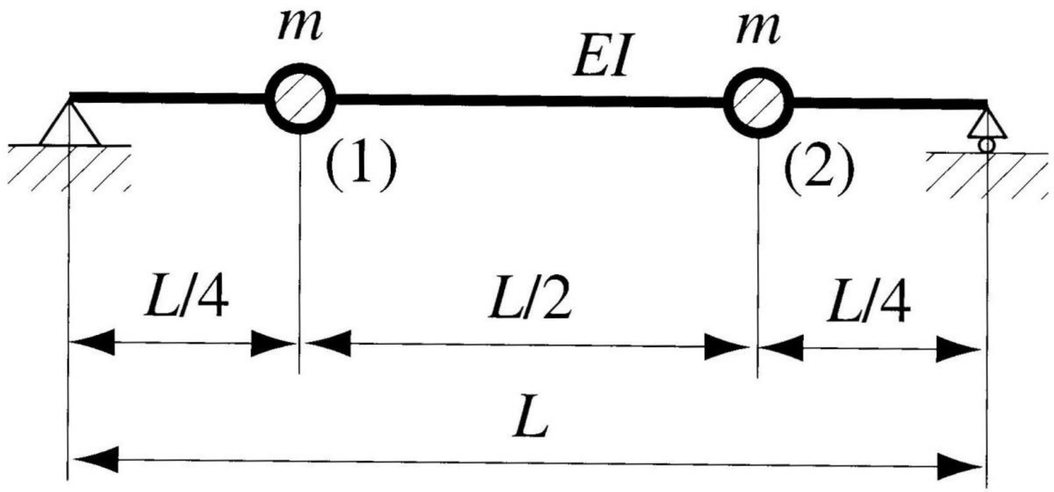

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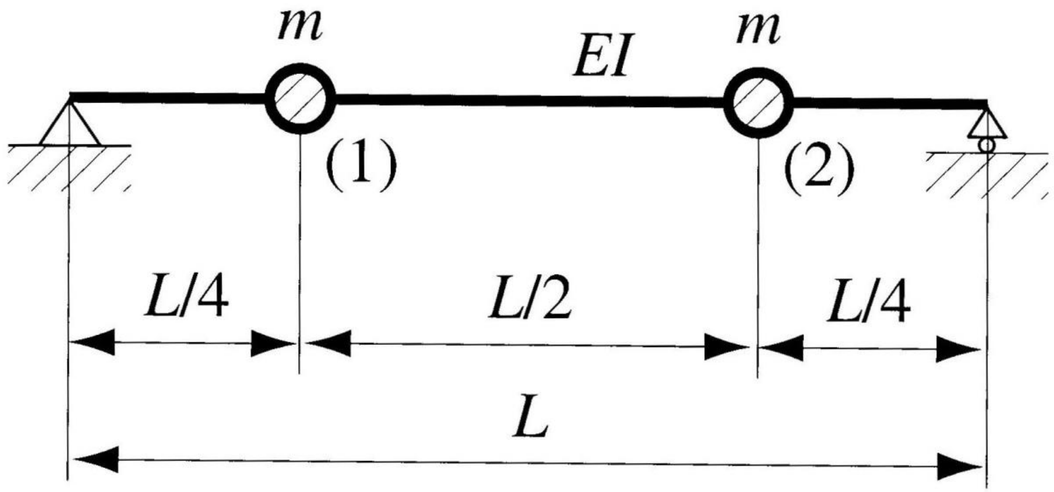
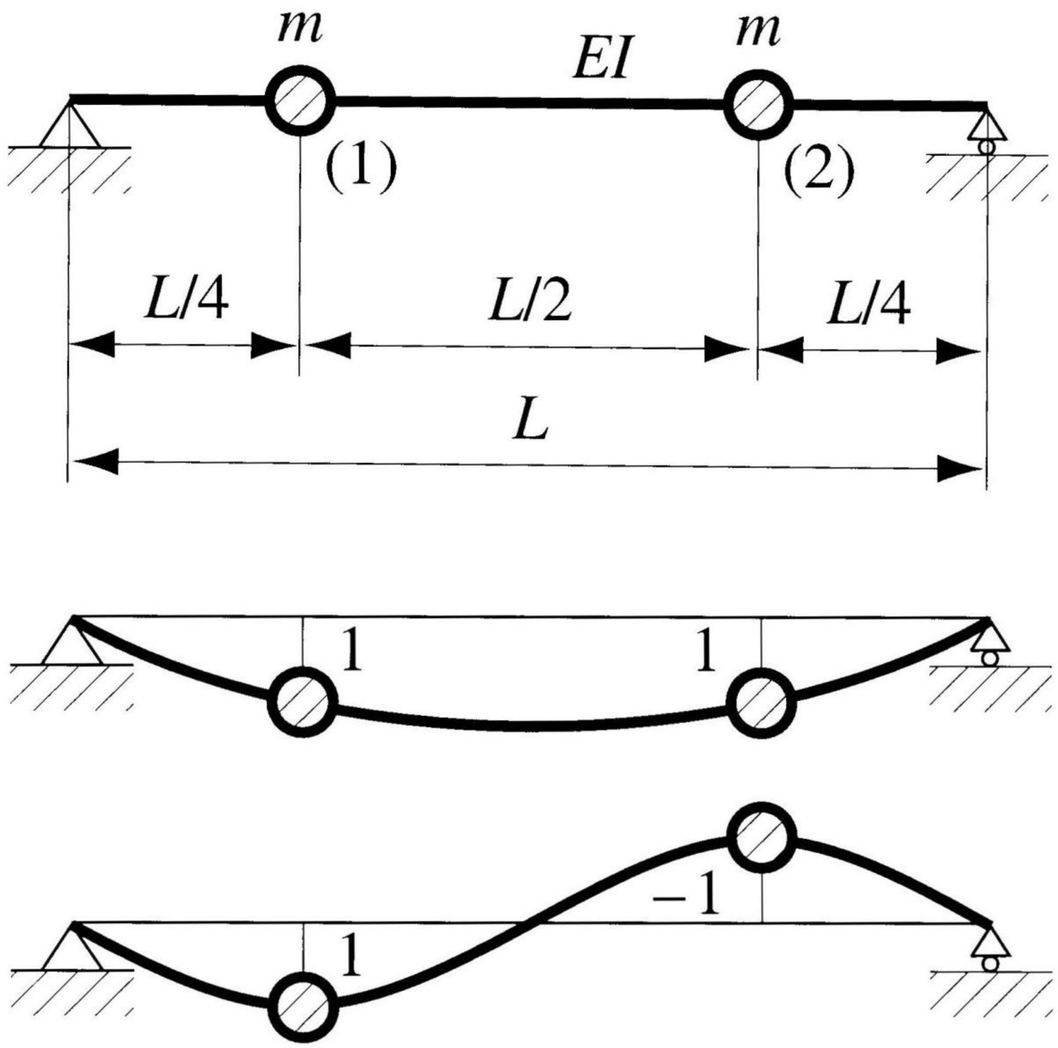


Exemple 8.2: Poutre supportant des masses

Vecteurs modaux de la structure

$$\begin{Bmatrix} X_{11} \\ X_{21} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = X_1 \boldsymbol{\beta}_1$$

$$\begin{Bmatrix} X_{12} \\ X_{22} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} = X_2 \boldsymbol{\beta}_2$$

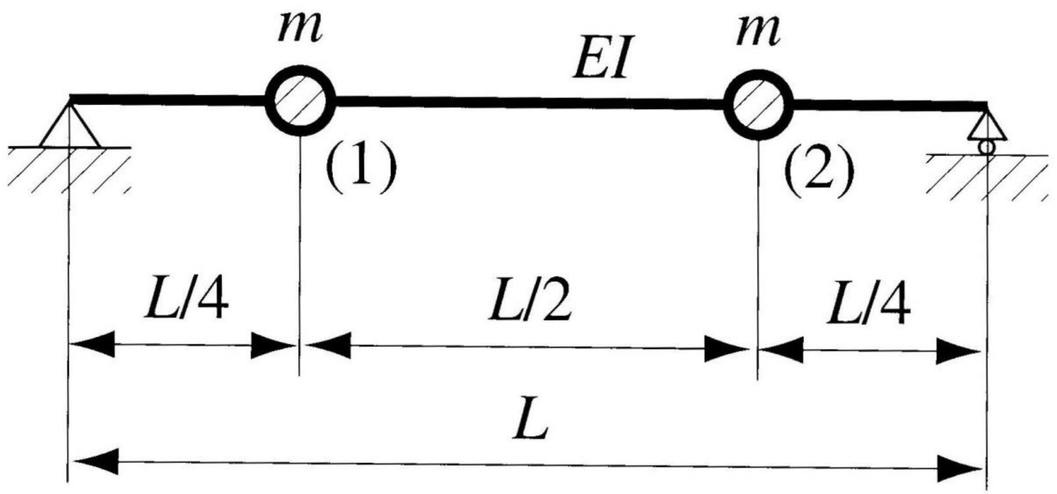


Exemple 8.3: Quotient de Rayleigh

Estimation des pulsations propres par le quotient de Rayleigh

$$\mathbf{K} = \alpha^{-1} = \frac{768EI}{L^3}\frac{1}{32}\begin{bmatrix} 9 & -7 \\ -7 & 9 \end{bmatrix} = \frac{24EI}{L^3}\begin{bmatrix} 9 & -7 \\ -7 & 9 \end{bmatrix}$$

$$M = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Exemple 8.3: Quotient de Rayleigh

Stationnarité du quotient de Rayleigh

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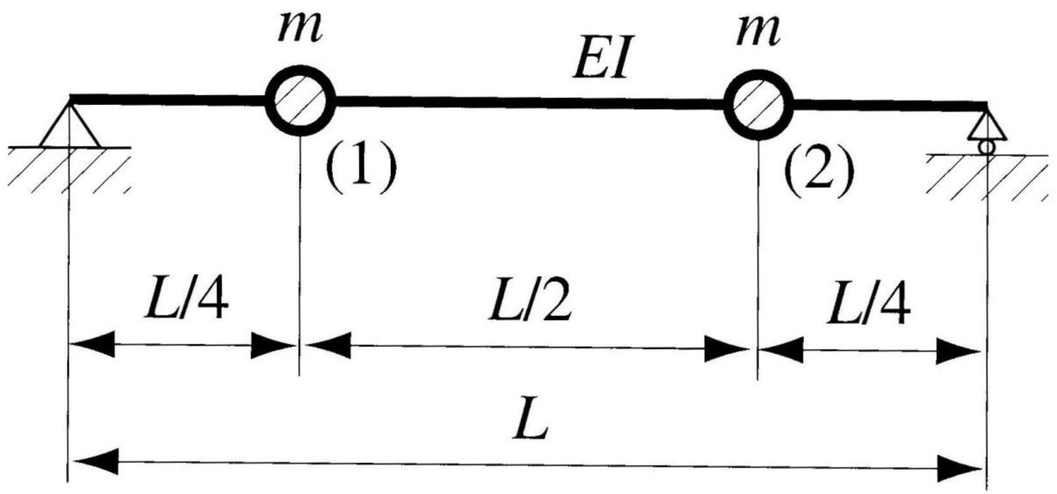
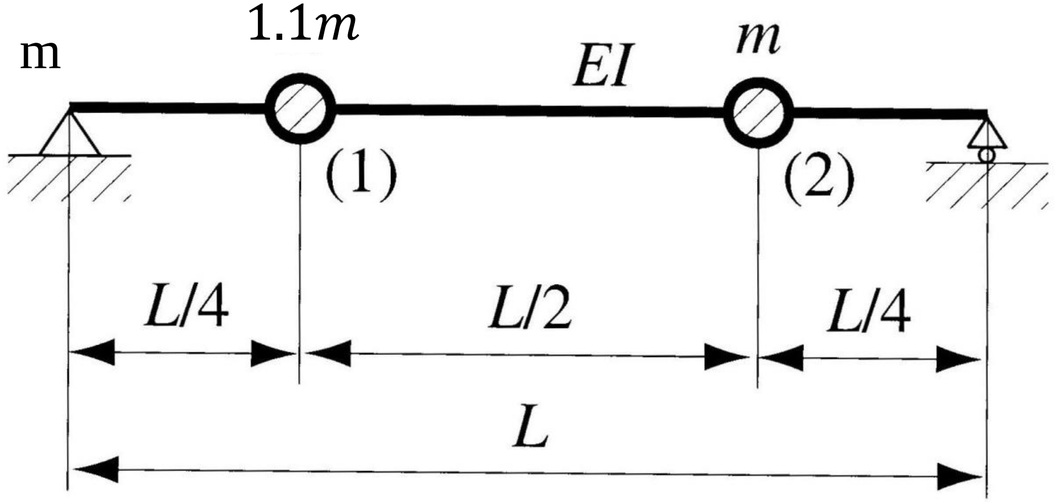


Exemple 8.3: Quotient de Rayleigh

On considère un petit défaut d'équilibrage des masses $\mu = 0.1$ m



$$\mathbf{M}_{new} = m\begin{bmatrix} 1+\mu & 0 \\ 0 & 1 \end{bmatrix} = m\begin{bmatrix} 1.1 & 0 \\ 0 & 1 \end{bmatrix}$$

Recherche d'une solution approchée

On sait que les modes propres seront ***<u>un peu différents</u>*** au cas symétrique. Alors, pour calculer les pulsations propres, on utilise la stationnarité du quotient de Rayleigh et on calcule R pour les vecteurs propres du cas symétrique:

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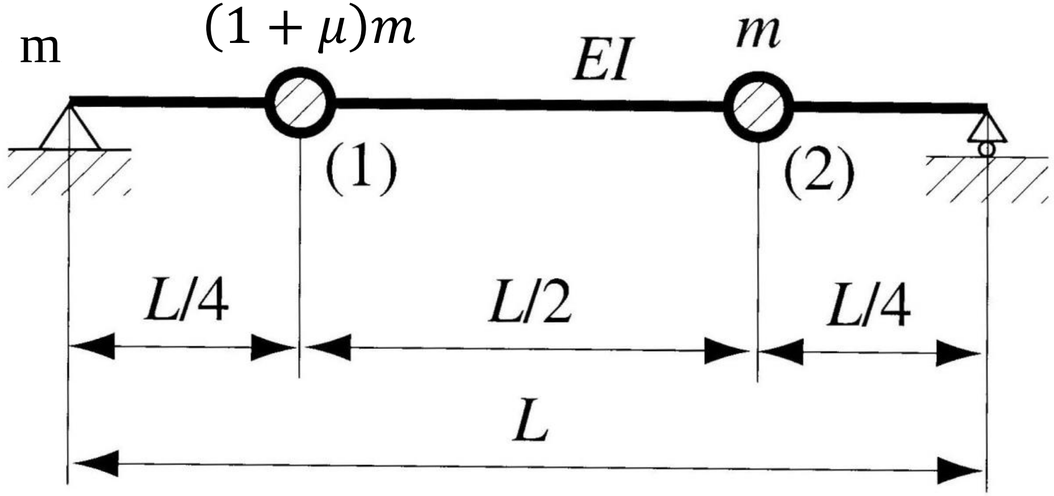
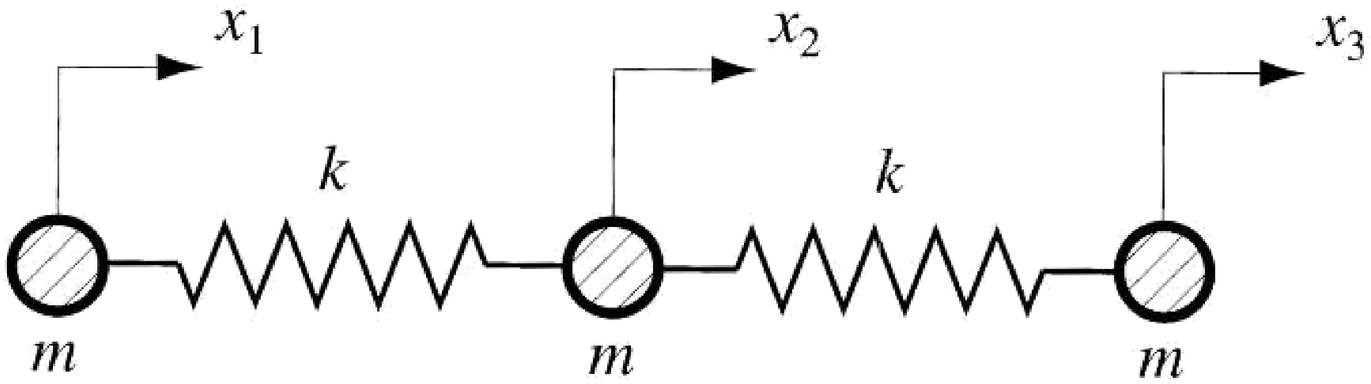


Mu	1ere Freq exacte	1ere Freq Rayleigh	Rel error (%)	2eme Freq exacte	2eme Freq Rayleigh	Rel error2 (%)
0	6.93	6.93	0.00	19.59591794	19.59591794	0.00
0.1	6.76	6.76	0.01	19.1468013	19.12328424	0.12
0.5	6.18	6.20	0.26	17.9387848	17.52712184	2.29
1	5.61	5.66	0.79	17.10263138	16	6.45
2	4.82	4.90	1.71	16.27267649	13.85640646	14.85
4	3.89	4.00	2.94	15.62049935	11.3137085	27.57

Exemple 8.4: 3 masses en mode libre-libre



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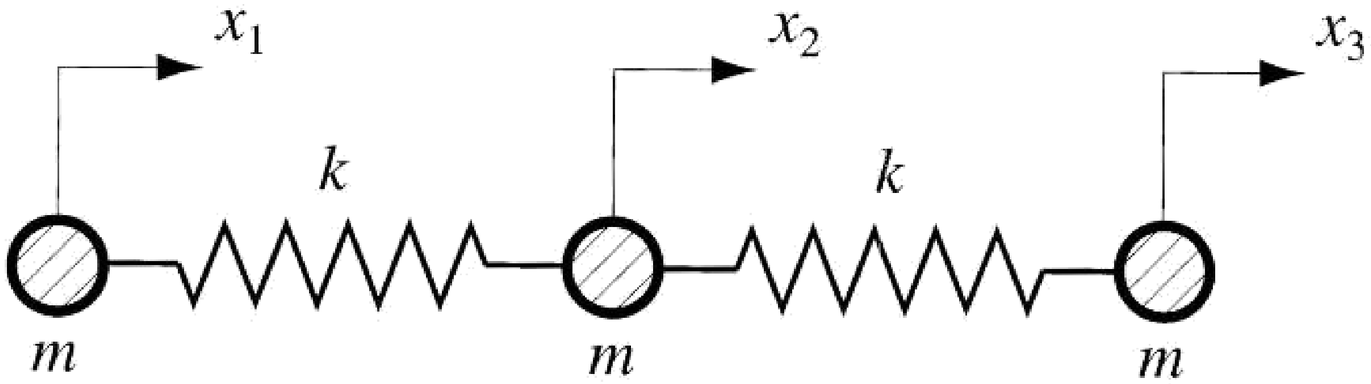

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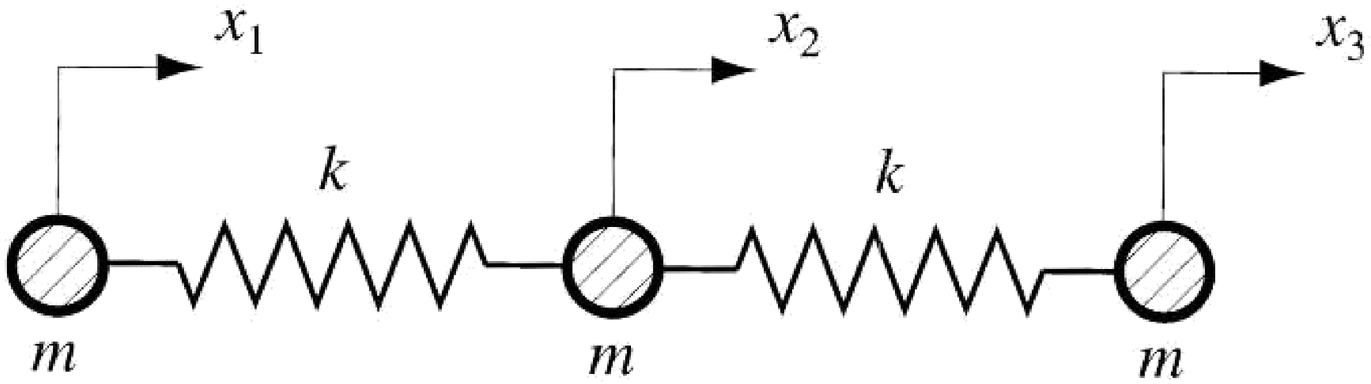

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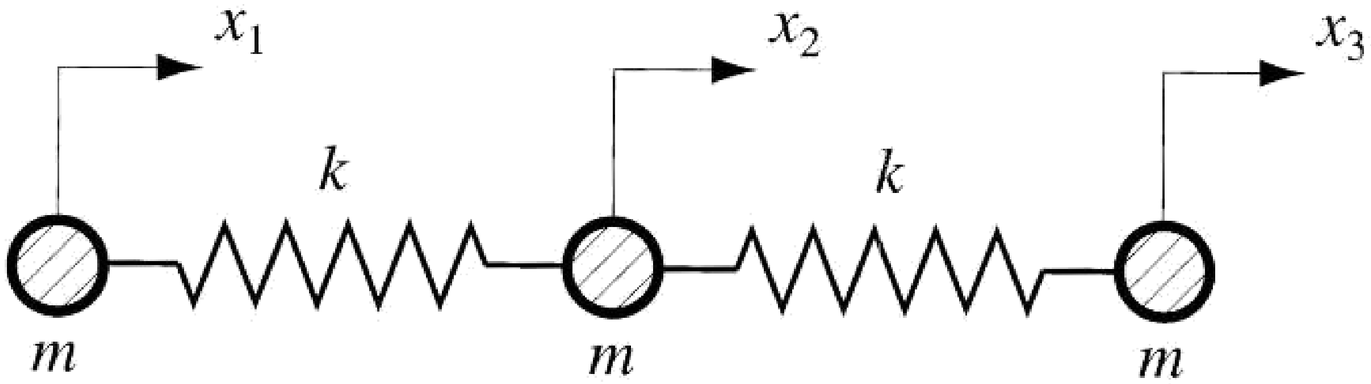

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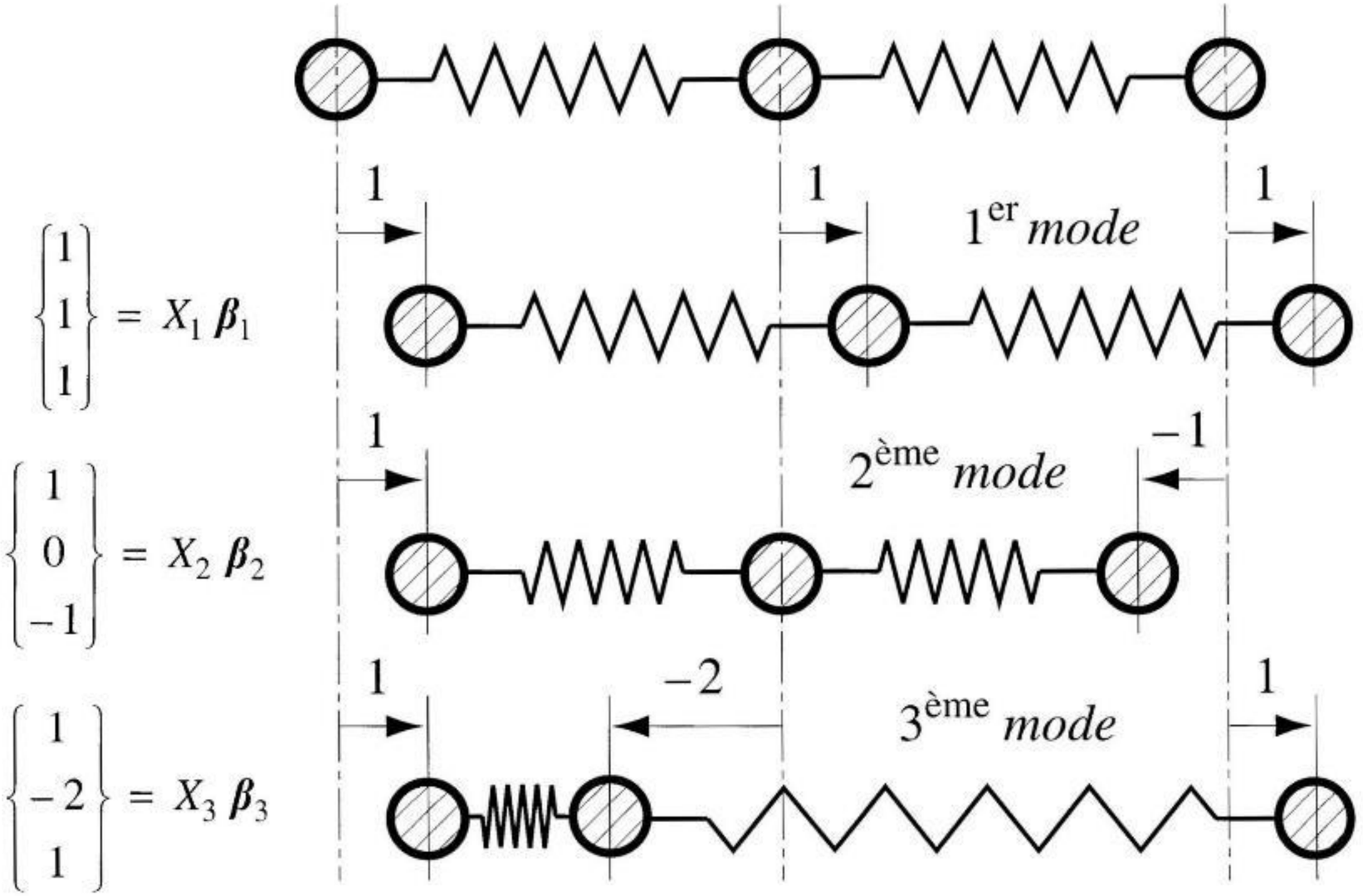

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